Attitude Control of a Satellite with Medium-Size Flexible Solar Arrays

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Theme

A CONTROLLER is designed for Canada's Communications Technology Satellite (CTS), a satellite with a large, flexible, "sun tracking" solar array. A linear transformation is used to isolate the characteristic modes of motion. The controller fires thrusters only when the states associated with rigid-body motion lie in sectors of the state space in which firings are "effective." In the CTS controller, array flexure states can be ignored; however, these states could be controlled if required.

Interesting aspects of the application are as follows: system eigenvalues are time-invariant despite a time-varying solar array angle $\gamma(t)$; a γ-dependent linear transformation provides a time-invariant Jordan canonical model and controller; only four states are used in both controller and observer; and γ is approximated by a constant midrange value. Modifications for more flexible craft are under study.

Contents

A model for CTS[1] is given in the backup paper. Three states, x_5 to x_7, represent the array flexure that has additional but unimportant modes. The other states, $x_1 \ldots x_4$, are associated with yaw and roll error and their derivatives. The parameter $\gamma(t)$ $(0 \leq \gamma \leq 90°)$ couples array motion to satellite attitude. There are control and disturbance inputs $u' = (\tau_{cr}, \tau_{cy})$ and $d' = (\tau_{dr}, \tau_{dy})$, respectively, and outputs $y' = (\phi, \psi)$ ($\phi \equiv$ and $\psi \equiv$yaw). Only ϕ is measured on CTS. We have the usual system equations:

$$\dot{x}(t) = A(t)x(t) + B(t)[u(t) + d(t)] \tag{1}$$

$$y(t) = C(t)x(t) \tag{2}$$

It is important that A can be partitioned as

$$A = \left[\begin{array}{c|c} A_{11} & 0 \\ \hline A_{21} & A_{22} \end{array}\right] \tag{3}$$

Because only the 3×4 A_{21} depends on γ, $|SI - A| = s(s^2 + \omega_1^2)(s^2 + \omega_2^2)[(s + \lambda) + \omega_3^2]$ is independent of γ, and there is a matrix $P(\gamma)$, having the same zero partition as A such that, if $x = Pz$, $\dot{z} = \Lambda z + P^{-1}B(u + d)$ and $y = CPz$, where Λ is Jordan real. For CTS, $\omega_1 = 7.27 \times 10^{-5}$ sec^{-1}, $\omega_2 = 1.806 \times 10^{-2}$ sec^{-1}, $\omega_3 = 2.3$ sec^{-1}, and $\lambda = 6.9 \times 10^{-3}$ sec^{-1}. The diagonal nature of Λ allows system motion to be studied in four subspaces: $z_1 \sim z_2$, $z_3 \sim z_4$, $z_5 \sim z_6$, and z_7. The first four rows of $P^{-1}B$ and $(CP)'$ ($'$ = transpose) are also independent of γ. The point is, if $z_5 \ldots z_7$ remain small, the system can be modeled as fourth-order time-invariant. Our controller fires the thrusters to keep the state projections on both the $z_1 \sim z_2$ and $z_3 \sim z_4$ planes small.

CTS uses hydrazine gas thrusters that are either "on" or "off." The net thrust reduces as propellant is used, and there is a minimum "turn-on" time of 10^{-3} sec. Our digital controller uses a sample period of 7.7 sec; firings occur, if at all, only during the last 10^{-3} sec of the sample period, and a square pulse of height 0.25 to 0.1 ft-lb is assumed. The minimum pulse duration maximizes pointing accuracy, and the end of the sample period firing time allows for compute times and ground loop control.

Unequal array areas and reflectivity result in a sinusoidal solar wind-induced disturbance torque[2] of amplitude of 0.75×10^{-5} ft-lb, with a 90° roll/yaw phase shift. These torques were assumed constant over the sample period and, for simulation studies, were generated using

$$w_{k+1} = \begin{bmatrix} \cos\omega_0 T & -\sin\omega_0 T \\ \sin\omega_0 T & \cos\omega_0 T \end{bmatrix} w_k \qquad \begin{aligned} \omega_0 &= 7.27 \times 10^{-5} \text{ sec}^{-1} \\ w_0' &= (0.75 \times 10^{-5}, 0) \\ T &= 7.7 \text{ sec} \end{aligned} \tag{4}$$

and

$$d_k = w_k + \zeta_k \tag{5}$$

where ζ_k, the random variation, was selected from a zero mean, gaussian, pseudorandom number generator (variance $= 0.75 \times 10^{-6}$).

An infrared "Earth sensor" produces a roll estimate once per second, with an error variance of 0.01°. This error was accounted for by a zero mean, white, gaussian noise term n_k, with variance 0.01° added to the value of roll at the observer input.

Defining $U = \{\Delta_k : \Delta_{ik} = \pm 1 \text{ or } 0,\ i = 1,2\}$, the satellite motion is given by a difference equation [$z_k \equiv z(kT)$, $k = 0,1,\ldots$; θ, H, and E are found in the usual way[2]]:

$$z_{k+1} = \theta z_k + H(\gamma)\Delta_k + E(\gamma)d_k \tag{6}$$

$$y_k = C(\gamma)P(\gamma)z_k \tag{7}$$

For CTS, measurement of roll alone does not suffice for the observation of all seven states,[3] and one also must measure yaw.[2] However, roll measurement suffices to observe z_i, $i = 1,2,\ldots 4$. We assume that states z_1 through z_4 dominate and truncate Eqs. (6) and (7). Let $z' = (z_1, \ldots z_4)$, θ be the upper-left-hand 4×4 corner of θ, and H coincide with the first four rows of H. Furthermore, assume that

$$z_{k+1} \simeq \theta z_k + \mathcal{H}\Delta_k \tag{8}$$

The motion of Eq. (8) is visualized best in the $z_1 \sim z_2$ and $z_3 \sim z_4$ planes. In each, if $\Delta_k = 0$, motion is circular about the origins with angular rates; $\omega_1 = 7.27 \times 10^{-5}$ and $\omega^2 = 1.806 \times 10^{-2}$ rad/sec for CTS. The $z_1 \sim z_2$ plane is

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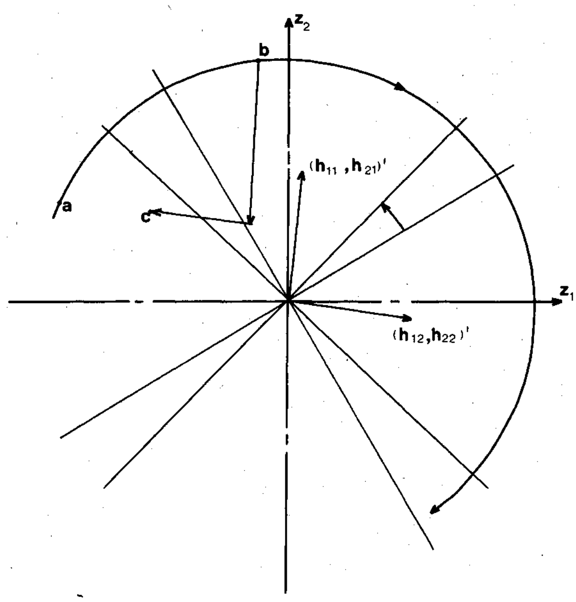

Fig. 1 Motion in z space.

illustrated in Fig. 1, where also are shown the projections $(h_{11}, h_{21})'$ and $(h_{12}, h_{22})'$ of the column vectors of $\mathcal{H}$. If the initial state is at point a, and if $\Delta_k = 0$, the new state will be at b; if, however, $\Delta_k \neq 0$, the new state can be found by adding to z_b the appropriate linear combination of the columns of $\mathcal{H}$. For example, if $\Delta_i = -1$ $(i = 1, 2)$ it will be at c.

Since a thruster firing implies a certain minimum movement in the two planes, there exists a set of irreducible states ∇. Given z_k, our controller calculates Δ_k^* such that $(\|z\|^2 \equiv \sum_{i=1}^{n} z_i^2)$,

$$\|z_{k+1}(\Delta_k^*)\|^2 = \min_{\Delta_k \in U} \{ \|z_{k+1}(\Delta_k)\|^2 \} \tag{9}$$

and it can be shown, if T is not chosen badly, that $\lim_{k \to \infty} z_k(\Delta_k^*) \in \nabla$. It is not always efficient, however, to fire the thrusters even if $\|z\|^2$ can be reduced; in fact, it is propitious to wait until z lies in certain sectors. With this in mind, we define a window $\mathcal{W}$ in both planes, and, after calculating Δ_k^*, the controller uses $\Delta_k = \Delta_k^*$ if $z_k \in \mathcal{W}$ and $\Delta_k = 0$ if not.

Two windows, $\mathcal{W}_1$ and $\mathcal{W}_2$, were studied. The first, $\mathcal{W}_1$, consists of four sectors of angular width β_1 and β_2 deg each, respectively, in each of the planes. Let h_1 and h_2 denote the two columns of $\mathcal{H}$. Then the sectors constituting $\mathcal{W}_1$ are centered on the projections of $\pm$ $(h_1 \pm h_2)$ (i.e., four per plane). The thrusters are fired according to Eq. (9) only is z_k lies in one of these sectors in both planes. β_1 and β_2 were chosen by experiment.

The second window, $\mathcal{W}_2$, consists also of four sectors as in $\mathcal{W}_1$; however, in this case they are centered on $\pm h_1$ and $\pm h_2$. An observer is used to provide an estimate $\mathcal{Z}_k$ of z_k for the controller. In the observer design, we assume that

$$z_{k+1} = \theta z_k + \mathcal{H}\Delta_k + \mathcal{E} w_k \tag{10}$$

where $\mathcal{E}$ coincides with the first four rows of $E(\gamma)$, and w_k is the assumed solar torque without random variations and is given by Eq. (4). The observer is described by

$$\mathcal{Z}_{k+1} = F\mathcal{Z}_k + \mathcal{H}\Delta_k + \mathcal{E} w_k + G\phi_k \tag{11}$$

Assuming $\phi_k \simeq \mathcal{C} z_k$, where $\mathcal{C}$ is a 1×4 formed from the first row of $C(\gamma)P(\gamma)$ and is the only control loop element dependent on γ, G can be chosen suitably to place the poles of $F = \theta - G\mathcal{C}^3$. For CTS, the poles of θ are $1 \pm j\, 5.6 \times 10^{-4}$ and $0.99 \pm j.139$. In our design, G was chosen to give F poles at $e^{-\alpha T}$ $(1 \pm j5.6 \times 10^{-4})$ and $e^{-\alpha T}$ $(0.99 \pm j.139)$; $\alpha = 1.1745 \times 10^{-3}$ gave good results.

Computer simulations were carried out to determine good window angles β_1 and β_2 and suitable values for the sample period T, the thruster pulse width δ, and the observer pole parameter α. System performance was assessed in the presence of random solar wind fluctuations, measurement noise, large initial errors both in the observer and in roll and yaw, and reductions in thrust. The effect of varying γ was tested, and pointing accuracy and fuel expenditures were estimated.

The solar wind was simulated by Eqs. (4) and (5), the satellite by Eqs. (6) and (7), the observer by Eq. (11) with ϕ_k replaced by $\phi_k + n_k$, and the controller used the observer estimate $\mathcal{Z}_k$ and the appropriate window $\mathcal{W}_i$, $i = 1,2$. Additional experiments are reported in Ref. 2.

To select β_1 and β_2, $w_k = 0$, and $\mathcal{Z}_k = z_k$ is assumed. Initial roll and yaw errors of 1.5° were chosen vice desired values of $< 0.1°$. Because motion in the $z_1 \sim z_2$ plane is slow, $\beta_1 = 90°$ was found to be a good choice. Smaller values for β_1 increased settling times. With $\beta_1 = 90°$, fuel usage increases with β_2, and a minimum settling time occurs for $\beta_2 \simeq 10°$-$20°$. The damping can be made similar to second-order critical by choice of β_2. The sample period T was varied from 1.3 to 11.7 sec without significant effect.

The thruster pulse width δ was varied from 0.01 to 0.05 sec. Longer pulses reduce both the settling time and the ultimate pointing accuracy. A variable pulse width scheme would be of advantage. Inclusion of the solar wind increases settling times by up to 15% and fuel usage by 8%.

Observer errors are of two types: "steady state," arising from both the exclusion of some states and the presence of ζ_k and n_k; and "transient," arising from the initial estimate error. In general, reducing α reduces steady-state errors but increases observer settling times. A compromise value of $\alpha \simeq 10^{-3}$ gave steady-state roll and yaw errors of about 0.02°, and an initial estimate error of 1° settles out in 1-2 min.

Experiments were carried out on the overall system: solar wind, satellite, observer, and controller. With window $\mathcal{W}_1$, the steady-state error remained less than 0.04° 86% of the time, whereas with $\mathcal{W}_2$ roll and yaw remained less than 0.04° 80% and 98.6% of the time, respectively. Use of $\mathcal{W}_1$ is estimated to require 3 lbm of fuel per year compared with only 1.8 for $\mathcal{W}_2$. Settling times were a little slower for $\mathcal{W}_2$. Decreased thrust increases settling times and improves pointing accuracy. Reducing thrust is equivalent to scaling down the elements of H. $\mathcal{H}$, however, was kept at its nominal value.

Several experiments were rerun with $\gamma = 0°$ vice 1 rad. That is, $\gamma = 0°$ was used in the model, with $\gamma = 1$ rad retained in the controller and observer equations. Settling times and fuel expenditures did not alter appreciably, although the pointing accuracy deteriorated noticeably.

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